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A Modified Stage-Stacking Method for Multi-Stage Axial Flow Compressor Calculations

Tonye K. Jack and Robin L. Elder

Abstract— In this paper an attempt was made at studying multi-sage axial flow compressor off-design performance based on two existing stage stacking correlations – the STEINKE (the United States NASA model – referred to as the NASA model), and the HOWELL-CALVERT (the United Kingdom NGTE model – refered to as the H-C model). Variable stator stagger setting is incorporated in an existing Cranfield optimisation model; and the primary objective is to see if improvements in overall compressor performance can be achieved by combining these models. An exhaustive study in multistage axial compressor performance has not been conducted. The authors however believe the relationships provide a basis at estimating axial compressor performance. A computer program based on the model will be developed.

Index Terms—Axial Compressor, Axial Performance, Compressor, Compressor Performance, Compressor Tip Clearance,, Multistage Compressor, Off-Design Performance, Stage Stacking, Variable Stator Stagger

1 Introduction

A Stage-to-stage analysis method is often adopted in evaluating and predicting the performance of multistage axial flow compressors from stage loading factor ($\Delta TC_p/U$) and pressure rise coefficient (ψ) versus flow coefficient (Φ) relationships [1]. Key parameters and operational considerations such as tip clearance, stage, stall, Reynolds Number, losses, and others are often designed and analysed based on certain existing derived analytical and/or experimental models. Several models exist. The poser for this research effort has been: So many Models! Can Axial Compressor Performance be improved by parameters study of combining related aspects of these several Models?

2 The Minimum Loss Model/Concept

2.1 Stalling Limits

The loss model adopted is based on the Wiscelenus [2] model, where, stalling or separation limits is defined based on the stalling coefficient, σ. In the Wiscelenus model, σ, has values of: σ = 0.5, 0.67, 1. The worst case scenario is assumed to occur at the minimum value of σ. This model was also adopted by Lieblein and others [3], in their break-through research work in deriving the NASA diffusion factors.

2.2 Flow Considerations

Based on the model adopted, limiting conditions where set for the stalling flow. Furthermore, critical rotor choke flow conditions can be predicted (see Appendix).

2.3 Effective Length

Figure (1) shows a simple model that was used to derive a relationship for locating stall point. Because of the stage interactions and mismatches that occur during flow separation, a reference location at entry to the stage with the first stage inlet as datum is used to indicate the stall point. This is effectively saying that since stall is "local", locating the point of stall could be anywhere after the exit of a stage plus the inlet to the next stage. This is similar to the work of Wassel [4], in which the effective length is indicated by the following relationship:

$$\text{Effective length} = \left(\frac{N}{N-0.5}\right)\begin{pmatrix}\text{entry tofirst stage rotor}\\ +\text{exit of last stage rotor}\end{pmatrix}$$

Where, N = stage number

This is similar to the present study in which the location of stall is indicated by the following relation:

Stall Point = (L_{eff} - L_x), with the datum at the inlet of the first rotor, and L_x calculated from the point of stall to the exit of last stage rotor.

2.4 Efficiency Correlation

In arriving at a useful relationship for the stall efficiency based on the H-C model [5], the assumption of stall point indicated by twice the minimum loss value, show that the factor, f = 1. This value indicates that the stall efficiency equals the maximum stage efficiency. This also implies that the H-C model applies only at flows above stall i.e. $M/M_s \geq 1$. An alternative approach will be a model that arrives at a lower, medium, and upper limiting values for the factor, f. This is a more useful model and shows that the stall efficiency is represented by the product: ($sef.\eta_{me}$). Where, sef is the stall efficiency factor based on the range of values of factor, f, when applied to "(6)" in the Appendix. The lower and upper limits of factor, f , correspondd to the stalling or separation limits. It should also be noted that the results arrived at agree with the inception of stall in the H-C correlation.

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3 Losses

The Koch-Smith [6] and Lakshiminarayana [7] models where incorporated in accounting for the effect of tip clearance, and other boundary layer effects. A general relationship for computing the tip clearance is given based on the Koch-Smith [6] model. At the stage maximum efficiency, the Lakshiminarayana [7] model was combined with the H-C [5] model, and provides a useful relationship for estimating the pressure loading, ψ, at maximum stage efficiency.

4 Annulus Blockage

A simple model was adopted to account for the effect of blockage. This model is based on the geometric annulus area and the overall displacement thickness (Appendix 5).

5 Stagger Angle Criterion

Effect of stagger setting can be studied with the model, by modifications of inlet and outlet angles, incidence and camber angles. However, the model is yet to be fully investigated in this direction and results compared to existing cranfield optimisation model [8] research that has been conducted in this area.

6 Effect of Reynolds Number

The effect of Reynolds Number has been built into the present model since the correlation for tip clearance, and displacement thickness calculations was based on the Koch-Smith [6] model. The Reynolds Number range is 2.5 x 10^5-10^7. Below this region is regarded as laminar.

4 Conclusion

This paper is not an exhaustive study in the Off-Design performance of multistage axial flow compressors. The objective of the exercise has been parameteric studies of the Steinke [9] and Howell-Calvert [5] models to arrive at useful relationships that can be applied in compressor design calculations. This modified method has not been applied to any existing compressors, and an example is therefore not presented. The authors however believe that the method can further be improved, including the derived equations with additional compressor parameters.

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Notations

P	Pressure, N/mm^2
T	Temperature, K
P_r	Pressure Ratio
T_r	Temperature Ratio
D	Diffusion Factor
deH	de Haller Number = 0.72
k_s	Specific heat Ratio
C_p	Specific heat at constant pressure, kJ/kg.K
R	Gas Constant, J/kg.K
M	Mass Flow, kg/s
M_n	Mach number
a	velocity of air, m/s
U	Blade speed, m/s
d	Rotor diameter, mm
s	space, mm
c	chord, mm
C_{ploss}	Profile loss coefficient
Re	Reynolds Number
ML	Minimum loss
N	Speed, rpm
A	Annulus area, m^2
B	Blockage factor
t	Tip clearance, mm
q	Relative temperature to relative flow
L_{eff}	Effective length, mm
L	Length in axial direction, mm
h	Blade height, mm
W	Work input
i	incidence angle
n	A factor
f	A factor
sef	Stall efficiency factor
V	Flow Velocity. m/s
F	Parameters

Subscripts and addition notation

a	Axial
s	indicates condition at stall
cr	Critical flow condition
me	Point of maximum efficiency
1	Inlet of compressor
x	Measured axially from compressor inlet stall location
NASA	National Aeronautics and Space Administration
NGTE	National Gas Turbine Establishment

Greek Symbols

ρ	Density, kg/m^3
η	Stage efficiency
η'	Stage efficiency without endwall losses
λ	Work done factor
δ^*	Displacement thickness, mm
$\delta_{F\theta}$	Tangential force thicjness, mm
κ	A constant
ζ	Stagger angle
θ	Comber angle
β_m	Mean flow angle
α_2	Air angle at outlet
α_1	Air angle at inlet
σ	Stalling coefficient factor
Ψ	Pressure coefficient

Φ	Flow coefficient
τ	parameter
τ_w	Shear Stress in wall
χ	constant
Λ	Degree of reaction
Δ	Difference or change in parameter

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APPENDIX

A.1 NORMAL FORM OF STEINKE MODEL

$$\phi = \frac{V_a}{U} \tag{1}$$

$$\Psi = \frac{C_p \eta T_1 (T_r - 1)}{U^2} \tag{2}$$

$$\eta = \frac{\left(P_r^{\left(k-1/k\right)} - 1\right)}{(T_r - 1)} \tag{3}$$

A.2 HOWELL-CALVERT MODEL

A.2.1 Stage Efficiency Relationship below Stall Flow

In this case: ($M/M_s<1$), and the required relationship is:

$$\frac{\eta}{\eta_s} = F_7\left(\frac{M}{M_s}\right) + (1 - F_7)\left(\frac{M}{M_s}\right)^n \tag{4}$$

A.2.2 Stage Efficiency Relationship above Stall Flow

In this case: ($M/M_s \geq 1$), and the required relationship is:

$$\frac{\eta}{\eta_{me}} = 1 - F_3 \frac{(q-1)^2}{q} \tag{5}$$

Where,

$$q = \frac{\left(\Delta T_{me} / \Delta T\right)}{\left(M / M_{me}\right)} \tag{5a}$$

Or

$$q = (\tan\alpha_1 - \tan\alpha_2) \tag{5b}$$

q≈1, at maximum stage efficiency

A.2.3 Stall Efficiency from Maximum Efficiency

$$\frac{\eta_s}{\eta_{me}} = 1 - \frac{5F_6(f-1)^2}{\left[1+23(f-1)^2\right]} \quad (6)$$

$$F_6 = \frac{(425+250\Lambda)}{500} \quad (6a)$$

A.2.4 Scale Effect

Because the H-C model was based on a diameter of 400 mm rotor stage, for other stages, an efficiency scaling factor given by “(7)” is applied:

$$\Delta\eta = 3\left(1-\frac{400}{d}\right) \quad (7)$$

A.2.5 Rotor Choke Influence

A correlation based on a change in temperature is used to express critical flow conditions and given by;

$$\tau = \frac{adjusted\Delta T}{original\Delta T} = 1 - F_4\left[\frac{0.01}{\left(1.01M/M_{cr}\right)}\right]^2 \quad (8)$$

$$corrected\,\eta = 1 - \frac{(1-original\,\eta)}{\tau} \quad (9)$$

$$F_4 = 0.008(-\zeta_{rotor}) \quad (10)$$

A.3 STEINKE WITH HOWELL-CALVERT

A.3.1 Specific Heat Ratio

From H-C model, at conditions other than air at k= 1.4, the following relationship applies:

$$k_s = \frac{U}{1.4T} \quad (11)$$

$$U = 1.4Tk_s \quad (12)$$

$$U^2 = 1.96T^2k_s^2 \quad (12a)$$

Substituting for U in “(2)” of the STEINKE model results in the following relationships:

$$k_s = 25C_p\eta\frac{(T_r-1)}{(49\Psi T)} \quad (13)$$

$$k_s = \left[25R\eta\frac{(T_r-1)}{(49\Psi T)}\right]+1 \quad (14)$$

A.3.2 Temperature Rise, ΔT_s

$$\Delta T_s = UV_a\left(\frac{\lambda}{C_p}\right)(\tan\alpha_1 - \tan\alpha_2) \quad (15)$$

After substitution through eliminating, U and C_p from “(15)”, the following modified relationship, “(16)” is obtained for the temperature rise:

$$\Delta T_s = \lambda\frac{(k_s-1)}{k_sR}\left(\frac{V_a^2}{\phi}\right)(\tan\alpha_1 - \tan\alpha_2) \quad (16)$$

Further substitution, results in a temperature rise, given by “(17)” in terms of the Mach number, M_n:

$$\Delta T_s = \lambda M_n^2\left(\frac{T}{\phi}\right)(k_s-1)(\tan\alpha_1 - \tan\alpha_2) \quad (17)$$

A.3.3 Pressure Rise Coefficient, ψ

$$\Psi = 25R\left[\frac{P_r^{\left(k_s-1/k_s\right)}-1}{49T(k_s-1)}\right] \quad (18)$$

The next couple of steps show methods for arriving at the stage efficiency relationships and other compressor parameters. It should be noted that major steps in their derivations have been omitted and only the key and final steps are shown.

A.4 Computations for Factor, *f*, and Efficiency

Factor *f*, in "(6)" from the H-C model is given by:

$$f = \frac{\left[stall : \left(\frac{\Delta V_w}{V} \right) \right]}{\left[\max .eff . : \left(\frac{\Delta V_w}{V} \right) \right]} \qquad (19)$$

The values for factor *f*, are, 0.5; 0.67; and 1 – based on the modified Wiscelenus loss relationship.

A.4.1 Stall Efficiency from Maximum Stage Efficiency

On assumption of 50 % reaction, the stall efficiency as a function of the maximum stage efficiency is given by:

For $f = 0.5$, $\eta_s = 0.8\eta_{me}$

For $f = 0.67$, $\eta_s = 0.829\eta_{me}$

For $f = 1$, $\eta_s = \eta_{me}$

A.4.2 Stage Efficiency from Maximum Stage Efficiency

From "(5)" of the H-C model,

$$\frac{\eta}{\eta_{me}} = 1 - \frac{F_3 (q-1)^2}{q} \qquad (20)$$

Substituting for *q*, gives "(21)":

$$\frac{\eta}{\eta_{me}} = 1 - \frac{F_3 \left[(\tan\alpha_1 - \tan\alpha_2) - 1 \right]^2}{(\tan\alpha_1 - \tan\alpha_2)} \qquad (21)$$

"(21)" can be reduced to the form of "(22)" by trigonometric adjustment,

$$\frac{\eta}{\eta_{me}} = 1 - \frac{F_3 \left[\tan(\alpha_1 - \alpha_2)(1+\kappa) - 1 \right]^2}{\tan(\alpha_1 - \alpha_2)(1+\kappa)} \qquad (22)$$

$$\alpha_1 - \alpha_2 = (-2\zeta + i + \theta) \qquad (23)$$

At maximum stage efficiency, $F_3=0$, and $\eta=\eta_{me}$

A.5 Displacement Thickness and Minimum Loss

A.5.1 Displacement Thickness

For turbomachines, the following relationships can be applied in estimating the displacement thickness, δ^*:

$$\delta^* = 1 - \int \rho V.dA \qquad (24)$$

$$Blockage = B = \left[1 - \left(1 - \sum \delta^* \right) \right] / A$$

The wall shear stress is:

$$\tau_w = 0.003 \rho V \qquad (25)$$

And the static pressure difference is given by,

$$\Delta P = 80 \tau_w \left(\frac{V}{\delta^*} \right) \qquad (26)$$

Substituting and rearranging, we have,

$$\frac{\Delta P}{\rho} = 0.24 V^2 \left(\frac{V}{\delta^*} \right) = W = mU.\Delta V_w \qquad (27)$$

A.5.2 Minimum Loss, ML

"(27)" can be rewritten to obtain "(28)"

$$\sigma \left[\frac{\Delta P}{\frac{1}{2}(\rho V)} \right] = \sigma (0.72) \left[\frac{2}{3} \left(\frac{V}{\delta^*} \right) \right] \qquad (28)$$

Where

$$ML \equiv \frac{\Delta P}{\frac{1}{2}(\rho V)} \qquad (28a)$$

A.5.3 Factor *n*

From "(4)",

$$\frac{\eta}{\eta_s} = F_7\left(\frac{M}{M_s}\right) + (1-F_7)\left(\frac{M}{M_s}\right)^n \qquad (29)$$

With F_7 typically taken as 0.5, the following relationship for factor *n* is obtained,

$$n = \frac{\left[Log\left(2\eta/\eta_s\right) - Log\left(M/M_s\right)\right]}{Log\left(M/M_s\right)} \qquad (30)$$

A.5.4 Computation of Stall Flow

Using figure (1), and the conditions for stall, the following limiting condition applies:

$$\frac{25}{24}\left(\frac{ML}{\delta^*\rho_x}\right) \le M_s \le \frac{25}{6}\left(\frac{ML}{\delta^*\rho_x}\right) \qquad (31)$$

A.5.5 Locating Stall Point

Again using figure (1), and noting that,

$$\frac{PV}{T} = constant \qquad (32)$$

Or

$$\frac{P_1 A_1 L_{eff}}{T_1} = \frac{P_x A_x L_x}{T_x} \qquad (32a)$$

Where,

$$L_x = \left[\frac{4(deH)}{3(ML)}\right]\left(\frac{P_1 A_1 L_{eff} T_x M_s}{A_x T_1 P_x \delta^* \rho_x}\right) \qquad (33)$$

Effective Stall Point = ($L_{eff} - L_x$) from datum.

A.6 Tip Clearance, Work Input into Stage

A.6.1 Tip Clearance

From the H-C model, the change in efficiency is given by:

$$\Delta\eta = 3\left[1 - \left(\frac{6280N}{7k_s T}\right)\right] \qquad (34)$$

or

$$\eta' - \eta = \Delta\eta = 3\left[1 - \left(\frac{6280N}{7k_s T}\right)\right] \qquad (34a)$$

Where, η' is the stage efficiency of the free stream without end-wall loss and given by the modified Koch-Smith relationship:

$$\eta = \eta'\left[\frac{\left(1 - \delta^*/h\right)}{\left(1 - \delta_{F\theta}/h\right)}\right] \qquad (35)$$

Upon substitution, and noting that, ($\delta_{F\theta}/t$) = 1.5, the tip clearance, *t*, is given by the relationship:

$$t = \frac{2}{3}h\left\{1 - \left\langle\left[\left(3 - \frac{(18840N)}{7k_s T}\right)\left(\frac{1}{\eta}\right) + 1\right]\left(1 - \delta^*/h\right)\right\rangle\right\} \qquad (36)$$

By combining the H-C model and the Lakshiminarayana loss estimating relationship,i.e.

$$\Delta\eta = \left[\frac{0.7\left(t/h\right)}{\cos\beta_m}\right]\left\{1 + 10\left[\frac{\phi\left(t/c\right)}{\Psi\cos\beta_m}\right]^{1/2}\right\} \qquad (37)$$

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The following simplified relationship for the tip clearance-to-chord ratio, (*t/c*), is obtained as:

$$\frac{t}{c} = (0.001)\Psi \cos \beta_m \qquad (38)$$

This can be used to estimate the loading coefficient at maximum efficiency.

A.6.2 Work Input into the Stage and Loss

From "(27)", the work input into the stage can be written as,

$$\frac{\Delta P}{\rho} = 0.24V^2\left(\frac{V}{\delta^*}\right) = W = mU.\Delta V_w = mUV_a(\tan\alpha_2 - \tan\alpha_1) \qquad (39)$$

Or

$$\frac{\Delta P}{(0.5\rho V^2)} = 0.48\left(\frac{V}{\delta^*}\right) = mV_a\left(\frac{U}{0.5V^2}\right)[\tan(\alpha_2 - \alpha_1)](1+\kappa) \qquad (40)$$

$$Loss = \frac{\Delta P}{(0.5\rho V^2)} = 0.48\left[\frac{V}{\delta^*}\right] = m\left(\frac{U}{0.5sV^2}\right)V_a[\tan(-2\zeta + i + \theta)](1+\kappa) \qquad (41)$$

Where, κ, is a constant given by,

$$\kappa = \frac{[\cos(\alpha_2 - \alpha_1) - \cos(\alpha_2 + \alpha_1)]}{[\cos(\alpha_2 - \alpha_1) + \cos(\alpha_2 + \alpha_1)]} \qquad (42)$$

A.7 Rotor Choke Condition

By combining the H-C and Koch-Smith models, the critical flow is given by,

$$M_{cr} = \frac{M}{\left[1.01 - F_4\left(0.01/\chi\right)\right]} \qquad (43)$$

Where,

$$\chi = 1 - \frac{1 - \left[3 - \left(18840N/7k_sT\right)\right]\left[\left(1 - \delta^*/h\right)/\left(1 - 3t/2h\right)\right]}{(1-\eta)} \qquad (44)$$

And

$$F_4 = 0.008\left[\frac{(\alpha_2 - \alpha_1 - i - \theta)}{2}\right] \qquad (45)$$

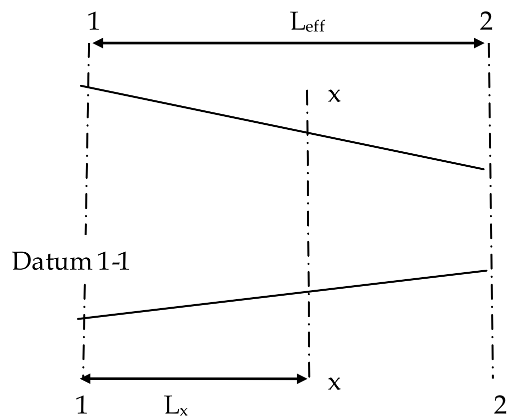

Fig.1: A simplified compressor model